CONTROLLER SETUP MAXIMIZING PERFORMANCE

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Abstract:
Most of available controllers has to be set up properly to obtain desired closed loop behavior. The searched controller is tuned to track a prescribed setpoint and fit specified constraints placed on its action variables. The tuning is proposed for LQG controller, for which the tuning is almost unexplored in compare to PID controllers. Tuning presented in this contribution is done for multidimensional adaptive LQG controller. Controller quality is evaluated by Monte Carlo approach using identified system model for performing off-line simulations. The numerical optimization searches the needed tuning.

Keywords: Adaptive Control, LQG Controller, Optimization

1. INTRODUCTION

This paper describes an approach of a multivariate controller design. Properly set up modern controller targets to achieve smaller disturbances and better desired value tracking of the controlled system output quantities than it is available with the classical controllers. Besides the control quality, it is also advantageous from the viewpoint of resources, energy, and production costs. The tuned controller is adjusted to fit to the specified constraints while considering the incomplete knowledge of the controlled system.

An important property of the majority of real systems is, that they have placed constraints on the variables, which must not be exceeded. The tuning parameters generally influence the effective range of controller variables, and so the tuning is a possible tool for ensuring the constraints compliance by the controller.

In the reality, the controlled system is never known completely. The aim of the work is to create a method of controller tuning which considers the model uncertainty. The controller is the adaptive one and it is tuned to satisfy the constraints in probabilistic sense according to the uncertainty of the model parameters. The controlled system is identified using Bayesian methodology, thus the model parameters are obtained in form of distribution, which describes the uncertainty.

2. CONTROLLER TUNING

The controller tuning process consists of the following parts. The identified system model to be controlled with inputs denoted by u_t and outputs by y_t at time t, is connected to the controller forming a closed loop. This configuration generates the closed loop data $d(T)$, consisting of both input and output values through whole simulation length T, see the left part of the Figure 1. The controller is parametrized by the searched tuning knobs quantity q.

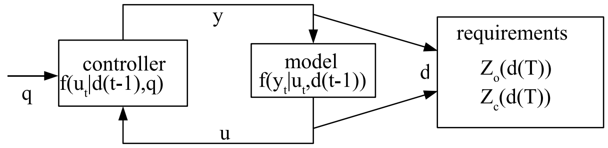

Fig. 1: Closed loop evaluation.

To evaluate the controller quality, the closed loop data $d(T)$ are used by loss functions Z_c and Z_o representing the users requirements fulfillment. The functions Z_c represents the constraint part and Z_o the objective part of the requirements, see the right part of the Figure 1. The task of the controller tuning is to find such tuning knob values, for which the requirements in the form of the loss functions are satisfied.

The dynamic stochastic system used for closed loop quality evaluation is given in the form of a pdf over the closed loop variables

$$f(y_t|u_t, d(t-1)). \tag{1}$$

This model is the result of the identification including uncertainty in parameters. The input actions u_t generated by a generally randomized controller are described by pdf

$$f(u_t|d(t-1), q), \tag{2}$$

where q denotes the tuning knobs. In fact, for most common deterministic controllers this pdf is defined as the Dirac function.

The constraints are described by data dependent function Z_c being non-positive, when the constraints are met

$$Z_c : d(T)^* \mapsto \mathbf{R}^{\mathring{c}}, \quad Z_c \leq 0, \tag{3}$$

where $\mathring{c}$ denotes the number of constraints. The controller performance objective function

$$Z_o : d(T)^* \mapsto \mathbf{R} \tag{4}$$

is decreasing with increasing controller performance.

The controller tuning is formulated as a stochastic optimization task

$$\begin{array}{rl} \text{minimize} & \mathcal{E}\{Z_o|q\} \\ \text{subject to} & \mathcal{E}\{Z_c|q\} \leq 0 \\ \text{over tuning knobs} & q \in q^*. \end{array} \tag{5}$$

3. CLOSED LOOP PERFORMANCE EVALUATION

3.1 Objective Function

The objective expresses commonly the wish that the quality of the regulation process in certain sense should be as high as possible subject to the present constraints. The desired signals setpoints are described by the d^{ref}. A typical wish on small tracking error of outputs and control effort of inputs is expressed by the objective function Z_o

$$Z_o^T = \frac{1}{T}\sum_{\tau=1}^{T}(d_\tau - d_\tau^{\text{ref}})' W (d_\tau - d_\tau^{\text{ref}}), \tag{6}$$

where W is a positive semi-definite matrix of appropriate dimensions.

The matrix W is usually diagonal with non-zero only those elements corresponding to signals in d with prescribed reference trajectory or setpoint. The values of the non-zero elements of W are usually chosen to be reciprocal to the variances of respective signals in d. This approach puts more importance on proper tracking of less noisy channels, while channels with higher variance take less effort of controller.

3.2 Noise Compensation Task

The second function evaluates a proportional amount of time where constraints are satisfied to the total length of simulation. In the discrete case, it is the relative frequency of constraints satisfaction

$$Z_{c,i}^T = \alpha_{\min} - \frac{1}{T}\sum_{t=1}^{T}\chi_{C_i}(d_{i,t}), \tag{7}$$

where χ_{C_i} is characteristic function of the set C_i defining the allowed data range, and number $\alpha_{\min} \in [0,1]$ relaxes the requirement of constraint satisfaction to a specified level.

This definition is suitable for situations where the constraints can be violated any time during the simulation. This is the case of noise compensation control, where the control loop generates a stationary process. Then it holds

$$Z_{c,i}^T \overset{T\to\infty}{\longrightarrow} \alpha_{\min} - \mathbf{P}(d_i \in C_i) \quad \text{almost surely}, \tag{8}$$

where $\mathbf{P}$ denotes probability.

4. OPTIMIZATION

In general case the pdf of data $f(d(T))$ is available only through samples. Thus the same holds for functions $Z_\bullet$. Therefore the optimization problem (5) forms a stochastic optimization task. Sample path method is used to approximate it by a deterministic optimization task.

Let for a function $h_\bullet : q^* \times \mathbf{R}^{\mathring{\xi}} \mapsto \mathbf{R}^{\mathring{Z}_\bullet}$ and random vector ξ hold

$$Z_\bullet(q) \equiv h_\bullet(q,\xi). \tag{9}$$

Let the expected value $\mathcal{E}Z_\bullet(q)$ be approximated with $\hat{Z}_\bullet(q)$

$$\hat{Z}_\bullet(q) = \frac{1}{N}\sum h_\bullet(q,\xi_i), \tag{10}$$

where N is a positive integer and $\{\xi_i\}_{i=1}^N$ is a sequence, called sample path, of independent samples of ξ. Fixing this sequence at constant samples, the optimization becomes deterministic. The deterministic optimization is solved by numerical optimization methods from Matlab Optimization Toolbox.

5. CLOSED LOOP COMPONENTS

5.1 Model

General description of the system is given by predictive pdf

$$\begin{aligned} f(y_t|u_t, d(t-1)) &= \\ &= \int\int f(y_t|u_t, \varphi_{t-1}, \theta, R) f(\theta, R|d(0)) d\theta dR, \end{aligned} \tag{11}$$

where y_t and u_t denotes system output and input; model state φ_t contains delayed signal values; input and output data up to time t are denoted by $d(t)$. The first factor in the integral in (11) is the ARX model of the system with paramers θ and variance R. The second factor in integral is the pdf of ARX model parameters obtained from Bayesian identification, see (Peterka, 1981). It is a Gauss-inverse-Wishart distribution. The integrated pdf $f(y_t|u_t, d(t-1))$ is the Student distribution being sampled during the simulation.

5.2 Controller

The controller action is deterministic $u_t = r(\varphi_t, q)$ and is generated by adaptive LQG controller. The LQG criterion is of the form

$$J_t = \sum_{\tau=t}^{t+T} (q_1 l_{1;\tau}^2 + q_2 l_{2;\tau}^2 + \ldots + q_n l_{n;\tau}^2), \tag{12}$$

where the scalar weights $q_\bullet \geq 0$, called penalization coefficients, are taken as the tuning knobs. The linear vector function l_t depends on quantities y_t, u_t and φ_t and measures the signal deviations from the desired values. Its elements are selected according to requirements on the controller. Common types are the tracking error $y_t - y_t^{\mathrm{ref}}$, magnitude of input signal u_t and its difference $u_t - u_{t-1}$.

6. EXPERIMENT

Presented experiment uses a model of single axis helicopter. The aim of the study is to create a controller respecting the given constraints placed on the action signal. The experiment uses adaptive controller to track the changing working point of the nonlinear Simulink model. For a comparison using the result is compared with non-adaptive controller behavior.

The aim of the control is to follow prescribed varying setpoint. The constraints are placed on the magnitude and increment of the action signal.

$$\begin{aligned} u_t &\in [-0.8, 0.8] \\ u_t - u_{t-1} &\in [-0.5, 0.5] \end{aligned}$$

6.1 Results

The tuned controller is defined by following criterion

$$J_t = \sum_{\tau=t}^{t+T} y_\tau^2 + q_1 u_\tau^2 + q_2 (u_\tau - u_{\tau-1})^2 \tag{13}$$

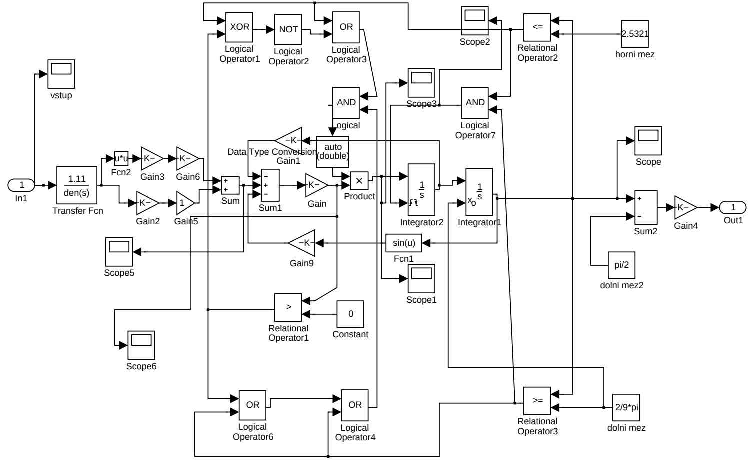

Fig. 2: Simulink scheme of the model

The optimized tuning knobs has value of $q = [0.034, 0.013]$.

The obtained controller is verified with the original Simulink model.. The result is shown in the Figure 6.1.

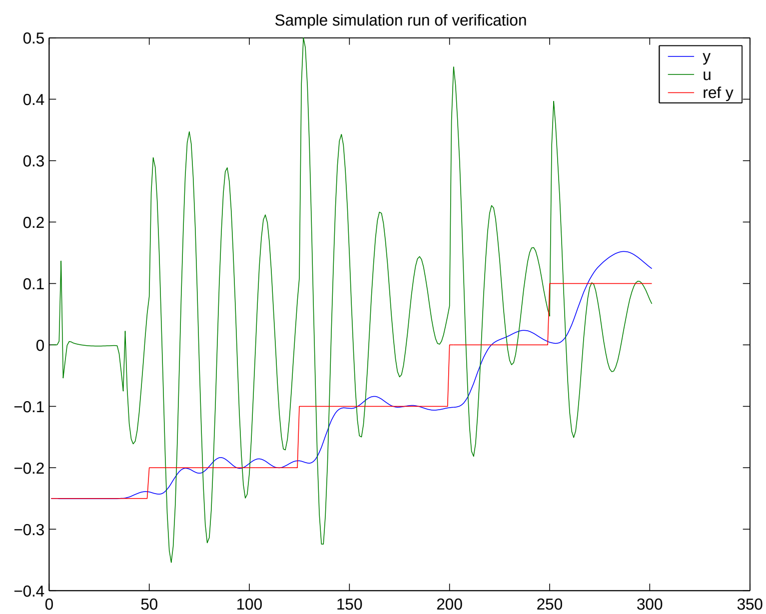

Fig. 3: Controller verification

The designed controller verified that action signal satisfies the prescribed constraints while keeping the output close to the varying reference trajectory. The adaptive property of the controller can be seen in the verification diagram, where the oscillation, caused by change of the working point of the nonlinear Simulink model, are being reduced as the model is being adapted for particular working point until it is changed to another value. This behavior demonstrates advantage of the adaptivity.

7. CONCLUSION

This contribution is focused on the adaptive LQG controller tuning. Multiple constraints and multiple input and output variables are considered. The method employs the Bayesian approach to deal with the uncertainty contained in imprecise knowledge of the controlled system. The obtained controller is calculated with respect to these uncertainty and takes on account whole range of uncertain parameters. The other method used are the Monte-Carlo for controller quality evaluation and numerical optimization for the tuning itself.

The results of this thesis shows that the aims where successfully achieved. The proposed methods and algorithms were implemented as a software toolbox Designer and successfully tested on several complex experimental models. The important contribution of the thesis is a step towards freeing the control engineers from manual controller tuning and to support usability of the model based controller in practice.

This method is being developed with the Designer Toolbox project (Novák *et al.*, 2003) in connection with the Jobcontrol environment (Tesař and Novák, 2005; Novák and Tesař, 2005) simplifying experiments tasks.

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